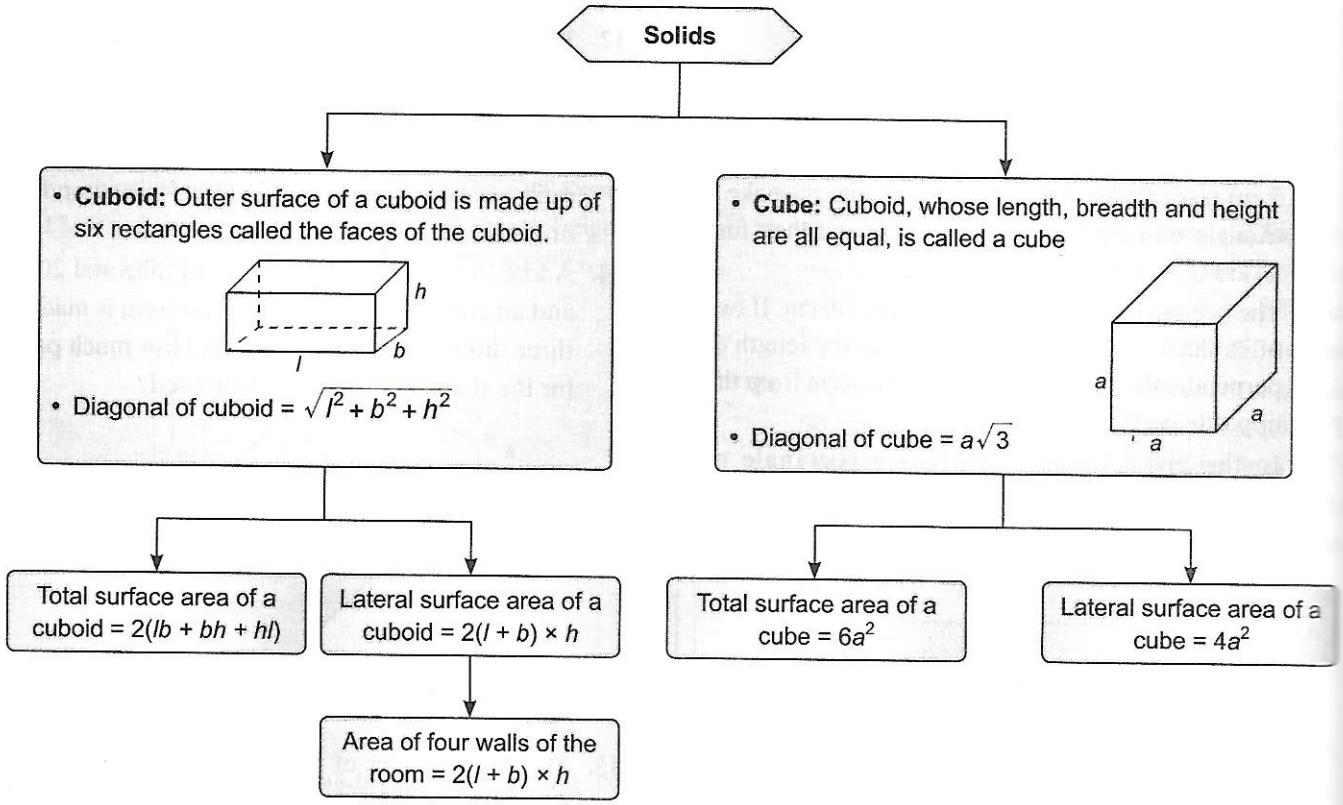
13 SURFACE AREAS AND VOLUMES

Surface Area of a Cuboid and a Cube



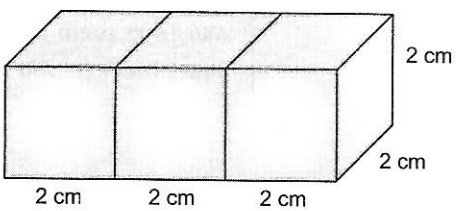
SOLVED QUESTIONS BASED ON EXERCISE 13.1

Very Short Answer Type Questions [1 Mark]

1. Three cubes are joined end to end forming a cuboid. If side of a cube is 2 cm, find the dimensions of cuboid thus obtained.

Sol. Given side of cube is 2 cm. Three cubes are joined end to end.

$\therefore$



Dimensions of cuboid thus formed is (2 + 2 + 2) cm, 2 cm, 2 cm, i.e. 6 cm, 2 cm, 2 cm.

2. Find the lateral surface area of cube, if its diagonal is $\sqrt{6}$ cm.

Sol. *Let edge of cube be l.*

$$\text{Diagonal of a cube} = l\sqrt{3}$$

$$\Rightarrow \quad l\sqrt{3} = \sqrt{6}$$

$$\Rightarrow \quad l = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2} \text{ cm}$$

$\Rightarrow$ Lateral surface area of a cube $= 4a^2 = 4 \times (\sqrt{2})^2 = 8 \text{ cm}^2$

3. The dimensions of the cuboid are 5 cm × 4 cm × 2 cm. Find its diagonal.

Sol. Here, $l = 5$ cm, $b = 4$ cm, $h = 2$ cm

$$\therefore \quad \text{Diagonal of a cuboid} = \sqrt{l^2+b^2+h^2} = \sqrt{5^2+4^2+2^2} = \sqrt{25+16+4} = \sqrt{45} \text{ cm}$$

$$= 3\sqrt{5} \text{ cm}$$

Short Answer Type Questions II [3 Marks]

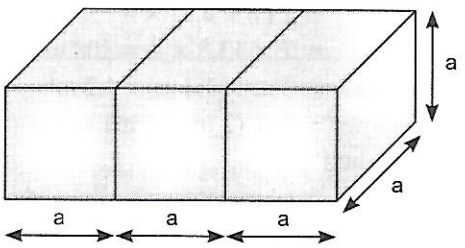
4. Three equal cubes are placed adjacent to each other in a row. Find the ratio of the total surface area of the cuboid thus formed to the total surface area of the three cubes.

Sol. Let edge of three equal cubes be a

Surface area of one cube $= 6a^2$

Total surface area of three cubes $= 3 \times 6a^2 = 18a^2$

Three equal cubes are joined end to end, a cuboid is formed as shown.



Length of resulting cuboid $= 3a$

Breadth of resulting cuboid $= a$

Height of resulting cuboid $= a$

Surface area of resulting cuboid $= 2(lb + bh + hl)$

$$= 2(3a \times a + a \times a + a \times 3a)$$

$$= 2(3a^2 + a^2 + 3a^2) = 14a^2$$

$$\therefore \quad \frac{\text{Total surface area of resulting cuboid}}{\text{Total surface area of three cubes}} = \frac{14a^2}{18a^2} = \frac{7}{9}$$

$\therefore$ Required ratio $= 7 : 9$

5. A wall paper 312 m long and 25 cm wide is required to cover the walls of a room. Length of the room is 7 m and its breadth is twice its height. Determine the height of the room.

Sol. Let height of room be h.

$\therefore$ Breadth $= 2 \times$ height $= 2h$ and length $= 7$ m

$\therefore$ Area of four walls of the room $= 2(l + b)\, h$

$$= 2(7 + 2h)\, h \qquad \ldots(i)$$

$\therefore$ Area of wall paper required to cover the four walls of the room $= 312 \times \left(\frac{25}{100}\right)$ ...(ii)

From (*i*) and (*ii*), we have $\frac{312 \times 25}{100} = 2(7 + 2h)\, h$

$\Rightarrow \quad \frac{312 \times 25}{100 \times 2} = (7 + 2h)\, h$

$\Rightarrow \quad 39 = (7 + 2h)\, h$

$\Rightarrow \quad 2h^2 + 7h - 39 = 0$

$\Rightarrow \quad 2h^2 + 13h - 6h - 39 = 0$

$\Rightarrow \quad 2h^2 - 6h + 13h - 39 = 0$

$\Rightarrow \quad 2h(h - 3) + 13(h - 3) = 0$

$\Rightarrow \quad (h - 3)(2h + 13) = 0$

$\Rightarrow \quad h = 3$ or $h = -\frac{13}{2}$ (neglected as height is never negative)

$\Rightarrow \quad h = 3$ m

$\therefore$ Height of the room = 3 m

Long Answer Type Questions [4 Marks]

6. A class room is 7 m long, 6.5 m wide and 4 m high. It has one door 3 m × 1.4 m and three windows each measuring 2 m × 1 m. The interior walls are to be colour washed. The contractor charges ₹ 5.25 per m^2. Find the cost of colour washing.

Sol. Given dimensions of classroom are $l = 7$ m, $b = 6.5$ m, $h = 4$ m

$\therefore$ Area of four walls of the room $= 2(l + b)h$

$= 2(7 + 6.5) \times 4$

$= 2 \times 13.5 \times 4 = 108$ m^2.

Area of one door $= 3$ m $\times 1.4$ m $= 4.2$ m^2

$\therefore$ Area of three windows $= 3 \times (2 \text{ m} \times 1 \text{ m}) = 3 \times (2 \text{ m}^2) = 6 \text{ m}^2$

Remaining area of walls to be colour washed

= (Area of four walls) – [Area of one door + Area of three windows]

$= 108 - (4.2 + 6)$

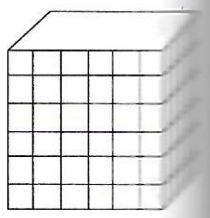
$= 108 - 10.2 = 97.8$ m^2

Cost of colour washing per m^2 = ₹5.25

Cost of colour washing 97.8 m^2 = ₹ 5.25 × 97.8 = ₹ 513.45

7. Hameed has built a cubical water tank with lid for his house, with each outer edge 1.5 m long. He gets the outer surface of the tank excluding the base, covered with square tiles of side 25 cm. Find how much he would spend for the tiles, if the cost of the tiles is ₹ 360 per dozen.

Sol. Since Hameed is getting the five outer faces of the tank covered with tiles, he would need to know the surface area of the tank, to decide on the number of tiles required.



Edge of the cubical tank = 1.5 m = 150 cm ($= a$)

So, surface area of the tank $= 5 \times 150 \times 150$ cm^2

Area of each square tile = side × side $= 25 \times 25$ cm^2

So, the number of tiles required $= \frac{\text{surface area of the tank}}{\text{area of each tile}} = \frac{5 \times 150 \times 150}{25 \times 25} = 180$

Cost of 1 dozen tiles, i.e., cost of 12 tiles = ₹ 360

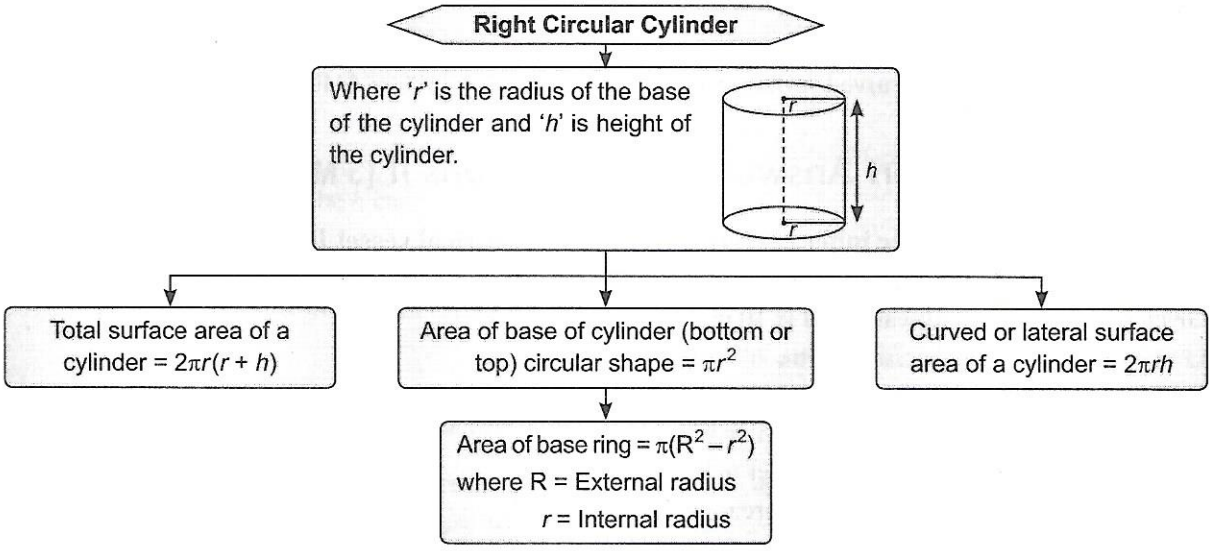
Therefore, cost of one tile = ₹ $\frac{360}{12}$ = ₹ 30

So, the cost of 180 tiles = 180 × ₹ 30 = ₹ 5400

PRACTICE QUESTIONS BASED ON EXERCISE 13.1

1. The length of the diagonal of a cube is $6\sqrt{3}$ cm. Find the length of its edge.
2. Find the length of the longest rod that can be put in a room of dimensions 10 m × 10 m × 5m. **[NCERT Exemplar]**
3. The perimeter of one face of a cube is 40 cm. Find the sum of the length of its edges. **[CBSE 2016]**
4. Length of a room is one and half times of its breadth. The cost of carpeting the room at ₹ 3.25 per m^2 is ₹ 175.50 and the cost of papering the walls at ₹ 1.40 per m^2 is ₹ 240.80. If 1 door and 2 windows occupy 8 m^2, find the dimensions of the room. **[HOTS]**
5. The cost of decorating the walls of a room at ₹ 6.60 per m^2 is ₹ 5082. The length and breadth of the room are in the ratio 4 : 3. Find the length and breadth of the room, if its height is 5.5 m.
6. The cost of preparing the walls of a room 12 m long at the rate of ₹1.35 per m^2 is ₹340.20 and the cost of matting the floor at 85 paise per m^2 is ₹91.80. Find the height of the room.
7. The dimensions of a rectangular box are in the ratio 2 : 3 : 4 and the difference between the cost of covering it with sheet of paper at the rate of ₹4 and ₹4.50 per m^2 is ₹416. Find the dimensions of the box.
8. The length, breadth and height of a room are 5 m, 4 m and 3 m respectively. Find the cost of colour washing its four walls and ceiling at the rate of ₹7.50 per square metre.

Surface Area of a Right Circular Cylinder



SOLVED QUESTIONS BASED ON EXERCISE 13.2

Short Answer Type Questions I [2 Marks]

1. The curved surface area of a right circular cylinder is 4400 cm^2. If the circumference of the base is 110 cm, find the height of the cylinder.

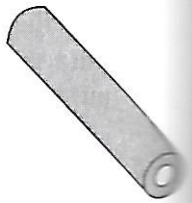
Sol. Let height of cylinder be h cm
and radius of cylinder be r cm
Given circumference of the base of cylinder $= 2\pi r = 110$ cm
and curved surface area of cylinder $= 4400\ cm^2$

$\Rightarrow \quad (2\pi r)\, h = 4400 \quad \Rightarrow \quad 110\, h = 4400$

$\Rightarrow \quad h = \dfrac{4400}{110} = 40$ cm

$\therefore$ Height of cylinder $= 40$ cm

2. Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. (see figure). What would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius? You may take $\pi = \frac{22}{7}$.



Sol. Radius of the base of the cylindrical kaleidoscope (r) = 3.5 cm

Height (length) of kaleidoscope (h) = 25 cm

Area of chart paper required = curved surface area of the kaleidoscope

$$= 2\pi rh = 2 \times \frac{22}{7} \times 3.5 \times 25 \text{ cm}^2 = 550 \text{ cm}^2$$

3. The inner diameter of a cylindrical vessel is 3.5 m. It is 100 m deep. Find the cost of polishing the inner curved surface area at the rate of ₹ 4 per m^2 $\left(\text{Use } \pi = \frac{22}{7}\right)$.

Sol. Given inner diameter of cylindrical vessel = 3.5 m

$\Rightarrow$ Inner radius of cylindrical vessel = $\frac{3.5}{2}$ m

$\Rightarrow$ Given depth of cylindrical vessel = 100 m

$\therefore$ Inner curved surface area of cylindrical vessel = $2\pi rh$

$$= 2 \times \frac{22}{7} \times \frac{3.5}{2} \times 100 = 1100 \text{ m}^2$$

Given cost of polishing per m^2 = ₹4

Total cost of polishing the curved surface area of 1100 m^2 = ₹4 × 1100 = ₹4400

Short Answer Type Questions II [3 Marks]

4. It costs ₹2,200 to paint the inner curved surface of a cylindrical vessel 10 m deep. If the cost of painting is at the rate of ₹20 per m^2, find the radius of the base. [CBSE 2010]

Sol. Given depth(h) of cylindrical vessel = 10 m

Let radius of cylindrical vessel be r m.

Curved surface area of cylindrical vessel = $2\pi rh$

$$= (2\pi r) \times 10 \text{ m}^2$$

Given cost of painting per m^2 = ₹20

Cost of painting inner curved surface area = ₹2200

$\therefore$ $20 \times (2\pi r) \times 10 = 2200$

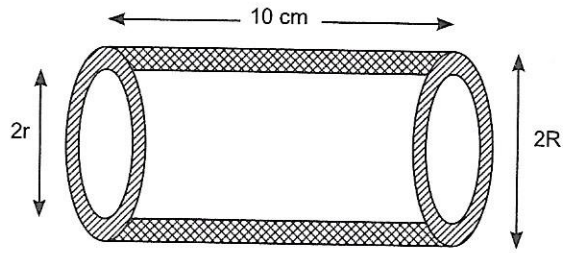
$\Rightarrow$ $20 \times 2 \times \frac{22}{7} \times r \times 10 = 2200$

$\Rightarrow$ $$r = \frac{2200 \times 7}{20 \times 2 \times 22 \times 10} = 1.75 \text{ m}$$

$\therefore$ Radius of base of a cylindrical vessel = 1.75 m.

5. The total surface area of a hollow metal cylinder open at both ends of external radius 8 cm and height 10 cm is 338 π cm^2. Taking r to be inner radius, find the thickness of the metal in the cylinder.

Sol.



Let r be the inner radius of a hollow metal cylinder.

Given external radius $= R = 8$ cm

and height $= 10$ cm

Total surface area of hollow metal cylinder = (Outer + Inner curved surface area of cylinder) + Area of base rings

$$338\pi = [2\pi Rh + 2\pi rh] + 2\pi(R^2 - r^2)$$
$$338\pi = 2\pi h\,[R + r] + 2\pi(R^2 - r^2)$$
$$338\pi = 2 \times \pi \times 10 \times [8 + r] + 2\pi(8^2 - r^2)$$
$$338\pi = \pi[20\,(8 + r) + 2(64 - r^2)]$$
$$338 = 160 + 20r + 2(64 - r^2)$$
$$\Rightarrow \quad 2r^2 - 20r + 338 - 160 - 128 = 0$$
$$\Rightarrow \quad 2r^2 - 20r + 50 = 0 \quad \Rightarrow \quad 2r^2 - 10r - 10r + 50 = 0$$
$$\Rightarrow \quad 2r(r - 5) - 10(r - 5) = 0 \quad \Rightarrow \quad (r - 5)\,(2r - 10) = 0$$
$$\Rightarrow \quad r = 5 \text{ cm}$$

$\therefore$ Thickness of the metal in the cylinder $= R - r = 8 - 5 = 3$ cm

Long Answer Type Questions [4 Marks]

6. Circumference of the base of a cylinder open at the top is 132 cm. The sum of radius and height of the cylinder is 41 cm. Find the cost of polishing the outer surface of cylinder at the rate of ₹10 per square metre.

Sol. Let radius of cylinder be r cm

Circumference of the base of the cylinder $= 2\pi r$

$$\Rightarrow \quad 2\pi r = 132$$
$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 132 \quad \Rightarrow \quad r = \frac{132 \times 7}{2 \times 22} = 21 \text{ cm}$$

Let height of the cylinder be h cm.

Also, $r + h = 41$ cm

$$\Rightarrow \quad 21 + h = 41 \quad \Rightarrow \quad h = 20 \text{ cm}$$

$\therefore$ Outer curved surface area of cylinder $= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 21 \times 20 = 2640 \text{ cm}^2$$

$\therefore$ Outer base area $= \pi r^2 = \frac{22}{7} \times 21 \times 21 = 1386 \text{ cm}^2$

Total outer surface area $= (2640 + 1386) = 4026 \text{ cm}^2$

Given cost of polishing per m^2 = ₹10

$\therefore$ Total cost of polishing $= \frac{₹10 \times 4026}{10000} = ₹4.026$

7. A cylindrical roller 2.5 m in length, 1.5 m in radius when rolled on a road was found to cover the area of 16500 m^2. How many revolutions does it make? **[CBSE 2014]**

Sol. Given radius of cylindrical roller $= 1.5$ m

and height of cylindrical roller $= 2.5$ m

$\therefore$ Area covered in one revolution = Curved surface area of cylinder

$$= 2\pi rh = 2 \times \frac{22}{7} \times 1.5 \times 2.5 \text{ m}^2$$

Let in 'n' number of revolutions, area covered is 16500 m^2.

Hence, $$n \times \left(2 \times \frac{22}{7} \times 1.5 \times 2.5\right) = 16500$$

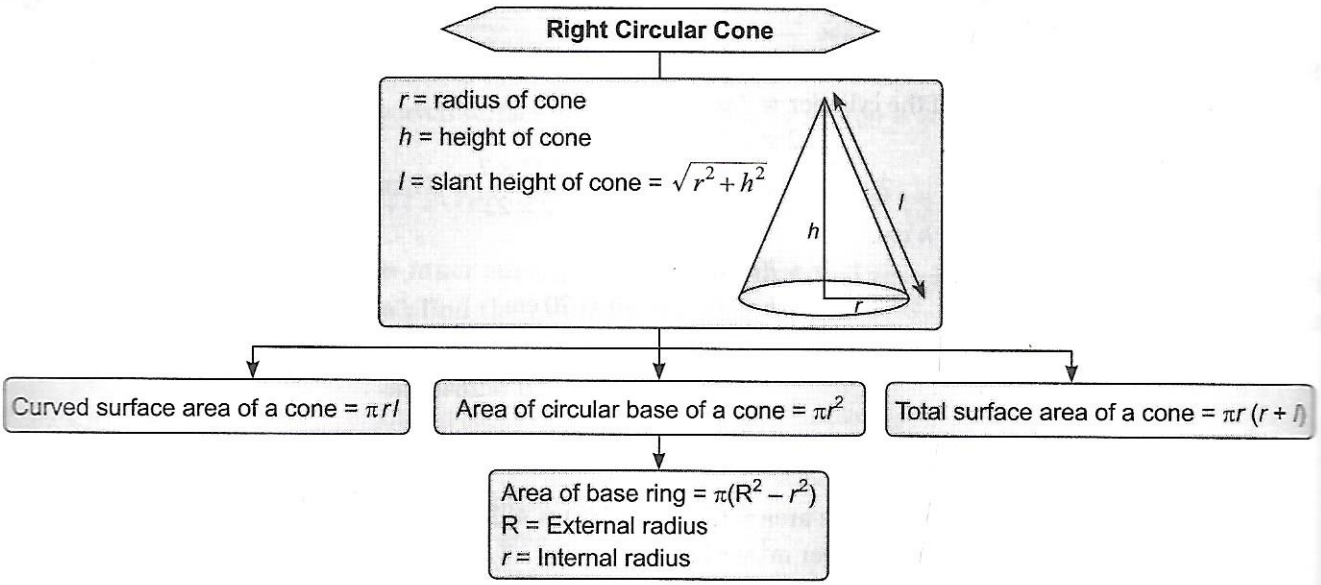
$$n = \frac{16500 \times 7}{2 \times 22 \times 1.5 \times 2.5} = 700$$

$\therefore$ A cylindrical roller makes 700 revolutions.

PRACTICE QUESTIONS BASED ON EXERCISE 13.2

1. A well 14 m deep is 2 m in radius. Find the cost of cementing the inner curved surface at the rate of ₹2 per square metre.
2. The total surface area of a solid right circular cylinder is 231 cm^2. If curved surface is two-third of the total surface area, determine the radius of its base and height.
3. The ratio between the curved surface area and total surface of a right circular cylinder is 1 : 2. Find the ratio between the height and radius.
4. An iron pipe 20 cm long has exterior diameter equal to 25 cm. If the thickness of the pipe is 1 cm, find the whole surface area of the pipe.
5. A factory manufactures 1,20,000 pencils daily. The pencils are cylindrical in shape, each of length 25 cm and circumference 1.5 cm. Determine the cost of colouring the curved surfaces of the pencils manufactured in one day at ₹0.05 per dm^2.
6. A metal pipe is 77 cm long. The inner diameter of a cross section is 4 cm and thickness of metal is 2 cm. Find its (*i*) inner curved surface area, (*ii*) outer curved surface area.
7. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Find the ratio of their total surface areas.

Surface Area of a Right Circular Cone



SOLVED QUESTIONS BASED ON EXERCISE 13.3

Very Short Answer Type Questions [1 Mark]

1. Find the total surface area of a cone whose radius is $\frac{r}{2}$ units and slant height is $2l$ units.

[NCERT Exemplar]

Sol. Here, Radius (R) = $\frac{r}{2}$ units, slant height (L) = $2l$ units

Total surface area of cone = π R (R + L)

$\therefore$ $= \pi\left(\frac{r}{2}\right)\left(\frac{r}{2}+2l\right) = \pi\left(\frac{r}{2}\right)\left(\frac{r+4l}{2}\right)$

$= \frac{1}{4}\pi r(r+4l)$ sq. units

2. Find the height of cone, if its slant height is 34 cm and base diameter is 32 cm.

Sol. Let height of the cone be h cm.

$$\text{Here, radius} = \frac{32}{2} = 16 \text{ cm}$$

$$\text{and slant height} = 34 \text{ cm}$$

$$\therefore \quad \text{Height of the cone} = \sqrt{l^2 - r^2} = \sqrt{(34)^2 - (16)^2}$$

$$= \sqrt{1156 - 256} = \sqrt{900} = 30 \text{ cm}$$

Short Answer Type Questions I [2 Marks]

3. How many square metres of canvas is required for a conical tent whose height is 3.5 m and the radius of whose base is 12 m?

Sol. Given radius (r) of the base of the cone = 12 m

and height (h) of the cone = 3.5 m

$\therefore$ Slant height (l) of the cone $= \sqrt{r^2 + h^2} = \sqrt{(12)^2 + (3.5)^2}$

$$= \sqrt{144 + 12.25} = \sqrt{156.25} = 12.5 \text{ m}$$

$\therefore$ Curved surface area of conical tent $= \pi rl$

$$= \frac{22}{7} \times 12 \times 12.5 = 471.42 \text{ m}^2$$

4. The diameters of two cones are equal. If their slant heights are in the ratio 7:4, find the ratio of their curved surface area. **[CBSE 2015]**

Sol. Let diameter of both cones be d.

Let radius $= \frac{d}{2} = r$ (say)

$\therefore$ Let slant height of first cone be $7x$ and slant height of second cone be $4x$.

Let C_1 and C_2 be curved surface area of first and second cone respectively.

$$\therefore \quad \frac{C_1}{C_2} = \frac{\pi r (7x)}{\pi r (4x)} = \frac{7}{4}$$

$$\Rightarrow \quad C_1 : C_2 = 7 : 4$$

$\therefore$ Ratio of their curved surface area = 7 : 4

Short Answer Type Questions II [3 Marks]

5. Anandita has a piece of canvas, whose area is 551 m^2. She used it to have a conical tent made with a base radius of 7 m. Assuming that all the stitching margins and the wastage incurred while cutting amount to 1 m^2. Find the slant height of conical tent.

Sol. Given radius of conical tent = r = 7 m

Let slant height of conical tent = l m

$\therefore$ Curved surface area of conical tent $= \pi rl = \frac{22}{7} \times 7 \times l = 22l \text{ m}^2$

Given area of canvas sheet = 551 m^2

While cutting and stitching, area of canvas wastage is 1 m^2.

$\therefore$ Area of canvas used for curved surface area of cone = 551 – 1 = 550 m^2

As per question,

$$550 = 22l \Rightarrow l = \frac{550}{22} = 25 \text{ m}$$

$\therefore$ Slant height of cone = 25 m

6. A corn cob (see figure), shaped somewhat like a cone, has the radius of its broadest end as 2.1 cm and length (height) as 20 cm. If each 1 cm^2 of the surface of the cob carries an average of four grains, find how many grains you would find on the entire cob.

Sol. Since the grains of corn are found only on the curved surface of the corn cob, we would need to know the curved surface area of the corn cob to find the total number of grains on it. In this question, we are given the height of the cone, so we need to find its slant height.

Here, $l = \sqrt{r^2 + h^2} = \sqrt{(2.1)^2 + 20^2}$ cm

$= \sqrt{404.41}$ cm $= 20.11$ cm

Therefore, the curved surface area of the corn cob $= \pi rl$

$= \frac{22}{7} \times 2.1 \times 20.11$ cm $= 132.726$ cm$^2 = 132.73$ cm^2 (approx.)

Number of grains of corn on 1 cm^2 of the surface of the corn cob $= 4$

Therefore, number of grains on the entire curved surface of the cob

$= 132.73 \times 4 = 530.92 = 531$ (approx.)

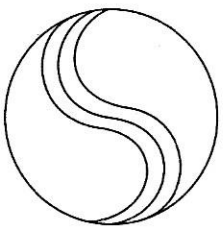
So, there would be approximately 531 grains of corn on the cob.

PRACTICE QUESTIONS BASED ON EXERCISE 13.3

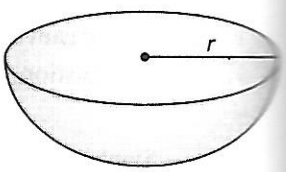
1. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.
2. The circumference of the base of 10 m high conical tent is 44 m. Calculate the length of canvas used in making the tent, if width of canvas is 2 m.
3. How many metres of cloth 1 m 10 cm wide, will be required to make a conical circus tent whose height is 12 m and the radius of whose base is 10 m? Also, determine the cost of the cloth at ₹ 7 per metre.
4. The radius and height of a cone are in the ratio 4 : 3. The area of the base is 154 cm^2. Find the area of the curved surface. (Use $\pi = \frac{22}{7}$)
5. Curved surface area of a cone is 308 cm^2 and its slant height is 14 cm. Find its total surface area. **[CBSE 2010]**
6. A conical tent is to accomodate 11 persons. Each person must have 4 square metres of space on the ground. What will be the curved surface area of conical tent, if its height is 15 m?
7. For Diwali celebration, a packet of 'ANARS' is bought. In that, there are 5 conical ANARS, with red glittering paper wrapped on it. If radius of the base of ANAR is 7 cm and slant height is 10 cm, find the total paper used to decorate the ANAR. **[HOTS]**

Surface Area of a Sphere and a Hemisphere

- **Sphere** : Let 'r' be the radius of the sphere.
  Surface area of a sphere $= 4\pi r^2$



- **Hemisphere** : Let 'r' be the radius of the hemisphere.
  Curved surface area of hemisphere $= 2\pi r^2$
  Total surface area of a hemisphere $= 2\pi r^2 + \pi r^2 = 3\pi r^2$



SOLVED QUESTIONS BASED ON EXERCISE 13.4

Very Short Answer Type Questions [1 Mark]

1. The surface area of two hemispheres are in the ratio 25 : 49. Find the ratio of their radii.

Sol. Let r and R be the radii of two hemispheres respectively.

$$\therefore \quad \frac{\text{Surface area of Ist hemisphere}}{\text{Surface area of 2nd hemisphere}} = \frac{25}{49}$$

$$\Rightarrow \quad \frac{3\pi r^2}{3\pi R^2} = \frac{25}{49}$$

$$\Rightarrow \quad \frac{r^2}{R^2} = \frac{25}{49}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{5}{7}$$

$\therefore$ Ratio of the radii of two hemispheres = 5 : 7

2. If surface area of a sphere is 676π cm^2, find its radius.

Sol. Let radius of sphere = r cm

Given surface area of sphere = 676π cm^2

$$\Rightarrow \quad 4\pi r^2 = 676\pi$$

$$\Rightarrow \quad 4r^2 = 676$$

$$\Rightarrow \quad r^2 = \frac{676}{4} = 169$$

$$\Rightarrow \quad r = 13 \text{ cm}$$

$\therefore$ Radius of a sphere = 13 cm

3. Mehul does not like the colour on the wooden ball he has. So he wants to scratch and remove the colour so that he can put the new one. How much area he has to scratch, if diameter of ball is r cm?

Sol. Given diameter of ball = r cm

$$\Rightarrow \quad \text{Radius of ball} = \frac{r}{2} \text{ cm}$$

$$\therefore \quad \text{Surface area of ball} = 4\pi\left(\frac{r}{2}\right)^2 = \frac{4\pi r^2}{4}$$

$$= \pi r^2 \text{ cm}^2$$

Short Answer Type Questions I [2 Marks]

4. Charvi took a spherical orange and put a thread around its boundary in the middle. She noted that the length of the thread was 22 cm. How much do you think was the diameter of the orange?

Sol. Let radius of spherical orange = r cm

$\therefore$ Circumference from the middle of spherical orange = Length of thread bound around the middle

$$\Rightarrow \quad 2\pi r = 22$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow \quad \frac{2r}{7} = 1 \quad \Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

$\therefore$ Diameter of the orange = $2r = 2 \times \frac{7}{2}$ cm = 7 cm

5. A dome of the building is in the form of hemisphere. From inside, it was white washed at the cost of ₹ 498.96. If the cost of white washing is ₹ 2 per square metre, find the inner surface area of dome.

Sol. Given cost of white washing per m^2 = ₹ 2

and cost of white washing curved surface area of dome = ₹ 498.96

$\therefore$ Curves surface area = $S = \frac{498.96}{2} = 249.48\ m^2$

Short Answer Type Questions II [3 Marks]

6. If the total surface area of sphere is 98.56 cm^2, find the radius of the sphere. [CBSE 2015]

Sol. Let radius of the sphere be r cm

Given surface area of sphere = 98.56 cm^2

$\Rightarrow \quad 4\pi r^2 = 98.56$

$\Rightarrow \quad r^2 = \frac{98.56 \times 7}{4 \times 22} = 7.84$

$\Rightarrow \quad r^2 = 7.84$

$\Rightarrow \quad r = 2.8$ cm

$\therefore$ Radius of the sphere = 2.8 cm

7. The outer curved surface areas of hemisphere and sphere are in the ratio 2 : 9. Find the ratio of their radii.

Sol. Let radius of sphere be R cm and radius of hemisphere be r cm.

$$\frac{\text{Outer curved surface area of hemisphere}}{\text{Surface area of sphere}} = \frac{2\pi r^2}{4\pi R^2}$$

$$\frac{2}{9} = \frac{2r^2}{4R^2}$$

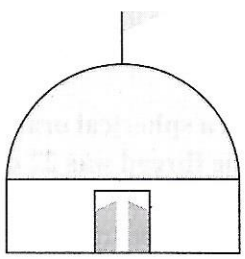
$\Rightarrow \quad \frac{r^2}{R^2} = \frac{4}{9}$

$\Rightarrow \quad \frac{r}{R} = \frac{2}{3}$

$\therefore$ Required ratio of their radii = 2 : 3

PRACTICE QUESTIONS BASED ON EXERCISE 13.4

1. The external and internal diameters of a hollow hemispherical vessel are 12 cm and 10 cm respectively. The cost of painting is ₹ 2 per sq. cm. Find the cost of painting the vessel all over.
2. Find the (*i*) curved surface area and (*ii*) total surface area of the hemisphere of radius 21 cm.
3. The hollow sphere, in which the circus motorcyclist performs his stunts has a diameter of 7 m. Find the area available to the motorcyclist for riding.
4. A hemispherical dome of a building needs to be painted. If the circumference of the base of the dome is 17.6 m, find the cost of painting it, given the cost of painting is ₹5 per 100 cm^2.



5. The diameter of sphere is decreased by 25%. By what percent does its curved surface area decrease?
6. The radius of hemispherical balloon increases from 6 cm to 12 cm as air is being pumped in to it. Find the ratio of surface area of balloon in the two cases. [NCERT Exemplar]

Volume of a Cube and a Cuboid

- Volume of a Cuboid = base area × height = length × breadth × height = $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid.
- Volume of a Cube = edge × edge × edge = a^3, where a is the edge of the cube.

SOLVED QUESTIONS BASED ON EXERCISE 13.5

Very Short Answer Type Questions [1 Mark]

1. The volume of cube is 216 cm^3. Determine its edge.

Sol. Let edge of cube be x cm.

Given volume of cube = 216 cm^3

$\Rightarrow$ $x^3 = 216$

$\Rightarrow$ $x = 6$ cm

2. From the given cuboid of dimensions l = 3 cm, b = 2 cm and h = 2 cm, how many cubes of edge 1 cm can be cut from it?

Sol. Given dimensions of cuboid are $l = 3$ cm, $b = 2$ cm, $h = 2$ cm

$\therefore$ Volume of cuboid $= l \times b \times h = 3 \times 2 \times 2$ cm^3

Given edge of cube = 1 cm

$\therefore$ Volume of cube $= a^3 = (1)^3 = 1$ cm^3

$\therefore$ $$\text{Number of cubes} = \frac{\text{Volume of cuboid}}{\text{Volume of cube}}$$

$$= \frac{3 \times 2 \times 2}{1} = 12$$

3. The length, breadth and height of a rectangular wooden box are 25 cm, 10.7 cm and 8.5 cm respectively. Find the volume of the box.

Sol. Given dimensions of a rectangular wooden box are $l = 25$ cm, $b = 10.7$ cm, $h = 8.5$ cm

$\therefore$ Volume of the box $= l \times b \times h$

$= 25 \times 10.7 \times 8.5$

$= 2273.75$ cm^3

Short Answer Type Questions I [2 Marks]

4. The volume of a committee room is 5760 m^3. Its length and breadth are 24 m and 20 m respectively. Find the height of the room.

Sol. Volume of committee room = 5760 m^3

Length of committee room = l = 24 m

Breadth of committee room = b = 20 m

Let height of committee room = h m

$\therefore$ Volume of committee room $= l \times b \times h$

$\Rightarrow$ $5760 = 24 \times 20 \times h$

$\Rightarrow$ $$h = \frac{5760}{24 \times 20} = 12 \text{ m}$$

$\therefore$ Height of the room = 12 m

5. Find the edge of a cube, if volume of the cube is equal to the volume of cuboid of dimensions 8 m × 4 m × 2 m.

Sol. Let edge of cube be x m.

$\therefore$ Volume of cube $= x^3$ m^3

Given dimensions of cuboid are $l = 8$ m, $b = 4$ m, $h = 2$ m

$\therefore$ Volume of cuboid $= l \times b \times h = 8 \times 4 \times 2 = 64$ m^3

$\therefore$ According to question, Volume of cube = Volume of cuboid

$\Rightarrow$ $x^3 = 64$

$\Rightarrow$ $x = (64)^{1/3}$

$\Rightarrow$ $x = 4$ m

$\therefore$ Edge of the cube = 4 m

Short Answer Type Questions II [3 Marks]

6. A reservoir is in the form of a rectangular parallelopiped (cuboid). Its length is 20 m. If 18 kl of water is removed from the reservoir, the water level goes down by 15 cm. Find the width of the reservoir. (1 kl = 1 m^3)

Sol. Length of rectangular parallellopiped (cuboid) = 20 m

Volume of water removed from reservoir = 18 kl

= 18 m^3 (1 kl = 1 m^3)

Level (height) of water in reservoir = 15 cm

$= \frac{15}{100}$ m

Let width of reservoir = b m

$\therefore$ Volume of reservoir $= l \times b \times h$

$\Rightarrow$ $20 \times b \times \frac{15}{100} = 18$

$\Rightarrow$ $b = \frac{18 \times 100}{20 \times 15} = 6$ m

$\therefore$ Width of the reservoir = 6 m

7. How many cubic centimetres of iron are there in an open box whose external dimensions are 36 cm, 25 cm and 16.5 cm, the iron being 1.5 cm thick throughout ? If 1 cubic cm of iron weighs 15 g, find the weight of the iron used in the box.

Sol. Given external dimensions of open box are L = 36 cm, B = 25 cm, H = 16.5 cm

$\therefore$ External volume of open box = L × B × H = 36 × 25 × 16.5

= 14850 cm^3

Given thickness of iron sheet throughout = 1.5 cm

$\therefore$ Internal length = l = 36 – (1.5 + 1.5) = 36 – 3 = 33 cm

Internal breadth = b = 25 – (1.5 + 1.5) = 25 – 3 = 22 cm

Internal height = h = 16.5 – 1.5 = 15 cm

Internal volume = $l \times b \times h$ = 33 × 22 × 15

= 10890 cm^3

$\therefore$ Volume of iron used = External volume – Internal volume

= (14850 – 10890) cm^3 = 3960 cm^3

Given 1 cm^3 of iron weighs = 15 g

$\therefore$ 3960 cm^3 of iron weighs = 15 × 3960 = 59400 g = 59.4 kg

8. A box with lid is made of 2 cm thick wood. Its external length, breadth and height are 25 cm, 18 cm and 15 cm respectively. How many cubic cm of liquid can be placed in it? Also, find the volume of the wood used in it.

Sol. Given external dimensions of box are L = 25 cm, B = 18 cm, H = 15 cm

$\therefore$ External volume of box = L × B × H = 25 × 18× 15

$= 6750 \text{ cm}^3$

Given thickness of wood = 2 cm

$\therefore$ Internal dimensions of box are $l = 25 - 4 = 21$ cm, $b = 18 - 4 = 14$ cm, $h = 15 - 4 = 11$ cm

$\therefore$ Volume of liquid that can be placed in box = Internal volume of box

$= l \times b \times h = 21 \times 14 \times 11 = 3234 \text{ cm}^3$

$\therefore$ Volume of wood used = External volume – Internal volume

$= 6750 - 3234 = 3516 \text{ cm}^3$

Long Answer Type Questions [4 Marks]

9. Three cubes of metal whose edges are in the ratio 3 : 4 : 5 are melted down into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

Sol. Given ratio of the edges of three cubes = 3 : 4 : 5

Let Edge of 1st cube = $3x$ cm

Edge of 2nd cube = $4x$ cm

Edge of 3rd cube = $5x$ cm

$\therefore$ Total volume of all the three cubes = $(3x)^3 + (4x)^3 + (5x)^3$

$= 27x^3 + 64x^3 + 125x^3$

$= 216x^3 \text{ cm}^3$

Let edge of new cube formed be y cm.

$\therefore$ Length of diagonal of new cube = $\sqrt{3}\, y$

$\Rightarrow$ $\sqrt{3}\, y = 12\sqrt{3}$ ($\because$ Given diagonal of new cube = $12\sqrt{3}$ cm)

$\Rightarrow$ $y = 12$ cm

$\therefore$ Volume of new cube = $(12)^3 \text{ cm}^3$

According to question, $216x^3 = (12)^3$

$\Rightarrow$ $216x^3 = 12 \times 12 \times 12$

$\Rightarrow$ $x^3 = \dfrac{12 \times 12 \times 12}{216} = 2 \times 2 \times 2$

$\Rightarrow$ $x^3 = 2 \times 2 \times 2$

$\Rightarrow$ $x = 2$ cm

$\therefore$ Edge of 1st cube = $3x = 3 \times 2 = 6$ cm

Edge of 2nd cube = $4x = 4 \times 2 = 8$ cm

Edge of 3rd cube = $5x = 5 \times 2 = 10$ cm

10. A wall was to be built across an open ground to cover a width (or breadth) of 10 m. The height of the wall is 4 m and thickness of the wall is 24 cm. If this wall is to be built up with bricks whose dimensions are 24 cm × 12 cm × 8 cm, how many bricks would be required?

Sol. Given dimensions of the wall are

Width (breadth) of wall = 10 m

Height of wall = 4 m

Thickness = 24 cm = $\dfrac{24}{100}$ m

Total space occupied by wall = Thickness × Width × Height = $\dfrac{24 \times 10 \times 4}{100} \text{ m}^3$

Given dimensions of brick are $l = 24 \text{ cm} = \dfrac{24}{100}$ m, $b = 12 \text{ cm} = \dfrac{12}{100}$ m, $h = 8 \text{ cm} = \dfrac{8}{100}$ m

$\therefore$ Space occupied by one brick = $l \times b \times h = \dfrac{24}{100} \times \dfrac{12}{100} \times \dfrac{8}{100} \text{ m}^3$

$$\therefore \quad \text{Number of bricks required} = \frac{\text{Space occupied by wall}}{\text{Space occupied by one brick}}$$

$$= \frac{24 \times 10 \times 4}{100} \times \frac{100 \times 100 \times 100}{24 \times 12 \times 8}$$

$$= 4166.66 \approx 4167 \text{ bricks}$$

11. The external length, breadth and height of a closed rectangular wooden box are 18 cm, 10 cm and 6 cm respectively and thickness of wood is 0.5 cm. When the box is empty, it weighs 15 kg and when filled with sand, it weighs 100 kg. Find the weight of 1 cm^3 of the wood and sand.

Sol. Given dimensions of closed wooden box are L = 18 cm, B = 10 cm, H = 6 cm

$\therefore$ External volume of box = L × B × H = 18 × 10 × 6 = 1080 cm^3

Given thickness of wood = 0.5 cm

$\therefore$ Internal dimensions of box are

$l = 18 - (0.5 \times 0.5) = 18 - 1 = 17$ cm, $b = 10 - (0.5 \times 0.5) = 10 - 1 = 9$ cm, $h = 6 - (0.5 \times 0.5) = 6 - 1 = 5$ cm

$\therefore$ Internal volume of box = $l \times b \times h = 17 \times 9 \times 5 = 765$ cm^3

$\therefore$ Volume of wood used = External volume – Internal volume

$= 1080 - 765 = 315$ cm^3

$\therefore$ 315 cm^3 of wood weighs = 15 kg

$$1 \text{ cm}^3 \text{ of wood weighs} = \frac{15}{315} = \frac{1}{21} \text{ kg}$$

When box is filled with sand, its total weight is 100 kg.

$\therefore$ Weight of sand only = Total weight of box when filled with sand – Weight of empty box

$= 100 - 15 = 85$ kg

$\therefore$ 765 cm^3 of sand weighs = 85 kg

$$1 \text{ cm}^3 \text{ of sand weighs} = \frac{85}{765} = \frac{1}{9} \text{ kg}$$

12. A teak wood log is cut first in the form of cuboid of length 2.3 m, width 75 cm and of certain thickness. Find its thickness, if its volume is 1.104 m^3. How many rectangular planks of size 2.3 m × 75 cm × 4 cm can be cut from the cuboid? [CBSE 2014]

Sol. Given dimensions of teak wood log (cuboidal shape) are $l = 2.3$ m, $b = 75$ cm $= 0.75$ m

Let thickness = t m

Given volume of teak wood log = 1.104 m^3

$\Rightarrow \quad l \times b \times h = 1.104$

$\Rightarrow \quad 2.3 \times 0.75 \times t = 1.104$

$$\Rightarrow \quad t = \frac{1.104}{2.3 \times 0.75} = 0.64 \text{ m}$$

Given dimensions of rectangular planks are $l = 2.3$ m, $b = 75$ cm $= 0.75$ m, $h = 4$ cm $= 0.04$ m

$$\therefore \quad \text{Number of rectangular planks} = \frac{\text{Volume of teak wood log}}{\text{Volume of rectangular planks}} = \frac{1.104}{2.3 \times 0.75 \times 0.04} = 16$$

13. A wall 6 m long, 5 m high and 0.5 m thick is to be constructed with bricks, each having length 25 cm, breadth 12.5 cm and height 7.5 cm. Find the number of bricks required to construct the wall, if it is given that cement and sand mixture occupy $\frac{1}{20}$ of the volume of the wall. [CBSE 2014]

Sol. Given dimensions of wall are $l = 6$ m, $b = 0.5$ m, $h = 5$ m

Volume of wall = $l \times b \times h = 6 \times 0.5 \times 5 = 15$ m^3

$$\text{Volume of cement and sand mixture} = \frac{1}{20} \text{ of Volume of wall} = \frac{1}{20} \text{ of } 15 = \frac{15}{20} = \frac{3}{4} \text{ m}^3$$

$\therefore$ Volume of bricks used $= 15 - \frac{3}{4} = \frac{57}{4}\ m^3 = 14.25\ m^3$

Given dimensions of brick are $l = 25$ cm $= 0.25$ m, $b = 12.5$ cm $= 0.125$ m, $h = 7.5$ cm $= 0.075$ m

Volume of one brick $= l \times b \times h = 0.25 \times 0.125 \times 0.075\ m^3$

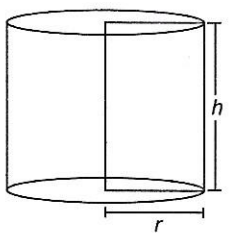
$$\text{Number of bricks} = \frac{\text{Volume of bricks used}}{\text{Volume of one brick}} = \frac{14.25}{0.25 \times 0.125 \times 0.075} = 6080$$

PRACTICE QUESTIONS BASED ON EXERCISE 13.5

1. The area of the bottom of a rectangular pit is 8.75 m^2. If its depth is 1.8 m, determine its volume.
2. One iron solid is a cuboid of dimensions 30 cm × 30 cm × 42.6 cm. It is melted and cubes each of side 3 cm are moulded from it. Find the number of cubes formed.
3. A pit of dimensions 2.5 m × 2.5 m × 4.5 m is dug at the rate of ₹ 15 per cubic metre. Find the cost of digging the pit.
4. Find the volume of a cube whose one of the diagonals is 2.5 m.
5. The area of three adjacent faces of a cuboid are x, y, z. If the volume is V, prove that $V^2 = xyz$.
6. A teak wood log is cut first in the form of a cuboid of length 2.5 m, width 0.69 m and of a certain thickness. Find its thickness, if its volume is 1.104 m^3. How many rectangular planks of size 2.5 m × 0.69 m × 0.08 m can be cut from the cuboid ?
7. A rectangular reservoir is 120 m long and 75 m wide. At what speed per hour must water flow into it through a square pipe of 20 cm wide so that the water rises by 2.4 m in 18 hours?
8. Water flows in a tank 150 m × 100 m at the base through a pipe whose cross section is 2 dm by 1.5 dm at the speed of 15 km/hr. In what time will the water be 3 m deep?
9. A rectangular container, whose base is a square of side 5 cm stands on a horizontal table and holds water upto 1 cm from the top. When a cube is placed in the water, it is completely submerged. The water rises to the top and 2 cubic cm of water overflows. Calculate the volume of the cube and also the length of its edge. **[HOTS]**
10. A rectangular tank measuring 5 m × 4.5 m × 2.1m is dug in the centre of the field measuring 13.5 m × 2.5 m. The earth dug out is spread evenly over the remaining portion of the field. How much is the level of the field raised? **[HOTS]**
11. A cube and a cuboid have the same volume. The dimensions of the cuboid are in the ratio 1 : 2 : 4. If the difference between the cost of polishing the cuboid and the cube at the rate of ₹ 5 m^2 is ₹ 80, find their volumes. **[HOTS]**
12. Find the number of cubes of side 2 cm that can be cut from a cuboid of dimensions 5 cm × 4 cm × 2 cm. **[CBSE 2016]**

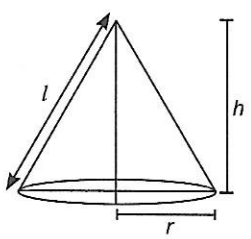
Volume of a Cylinder and a Cone

- Volume of a Cylinder $= \pi r^2 h$,
  where r is the base radius and h is the height of the cylinder.



- Volume of a Cone $= \frac{1}{3}\pi r^2 h$

  where r is the base radius, h is the height of the cone and l is the slant height of the cone.



SOLVED QUESTIONS BASED ON EXERCISES 13.6 AND 13.7

Very Short Answer Type Questions [1 Mark]

1. The radii of two cylinders are in the ratio 2 : 3 and their heights are in the ratio 5 : 3. Find the ratio of their volumes. [NCERT Exemplar]

Sol. Let radii of Ist cylinder = $2x$ and radii of 2nd cylinder = $3x$

Let height of Ist cylinder = $5y$ and height of 2nd cylinder = $3y$

$$\therefore \quad \text{Ratio of their volumes} = \frac{\frac{1}{3}\pi(2x)^2 \times 5y}{\frac{1}{3}\pi(3x)^2 \times 3y} = \frac{4x^2 \times 5y}{9x^2 \times 3y}$$

$$= \frac{20x^2y}{27x^2y} = \frac{20}{27} = 20 : 27$$

2. If the radius of right circular cone is halved and its height is doubled, then the volume will remain unchanged. Is it true or false? Justify your answer. [NCERT Exemplar]

Sol. Let radius of cone be r cm and height of cone be h cm.

$$\therefore \quad \text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

Now, $$\text{New radius} = \frac{r}{2} \text{ cm, New height} = 2h \text{ cm}$$

$$\therefore \quad \text{New volume of cone} = \frac{1}{3}\pi\left(\frac{r}{2}\right)^2 \times (2h)$$

$$= \frac{1}{3}\pi \times \frac{r^2}{4} \times 2h = \frac{1}{2}\left(\frac{1}{3}\pi r^2 h\right)$$

Clearly, the volume will change as the new volume is half of the original volume. Hence, the given statement is false.

3. A student has rectangular sheet of dimensions 14 cm × 22 cm. He wants to make a cylinder in such a way so that volume is minimum. Find the height of cylinder. [HOTS]

Sol. Volume will be minimum, when rectangular sheet is rolled along minimum length, i.e. when sheet is rolled along 14 cm. In this case, height will be 22 cm.

Short Answer Type Questions I [2 Marks]

4. The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the volume of cylinder.

Sol. Let radius of cylinder = r cm

$$\text{Height of cylinder} = 14 \text{ cm}$$

$$\text{Curved surface area of cylinder} = 88 \text{ cm}^2$$

$$\Rightarrow \quad 2\pi rh = 88$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 88$$

$$\Rightarrow \quad r = \frac{88 \times 7}{2 \times 22 \times 14} = 1 \text{ cm}$$

$$\therefore \quad \text{Volume of cylinder} = \pi r^2 h = \frac{22}{7} \times 1 \times 1 \times 14 = 44 \text{ cm}^3$$

5. Determine the volume of a conical tin having radius of the base as 30 cm and its slant height as 50 cm. (Use $\pi = 3.14$) [CBSE 2010]

Sol. Radius of base of conical tin $= r = 30$ cm

Slant height of conical tin $= l = 50$ cm

$\therefore$ Height of conical tin $= h = \sqrt{l^2 - r^2} = \sqrt{(50)^2 - (30)^2}$

$= \sqrt{2500 - 900} = \sqrt{1600} = 40$ cm

$\therefore$ Volume of conical tin $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 30 \times 30 \times 40$

$= 314 \times 3 \times 40 = 37680 \text{ cm}^3$

6. The volume of a cylindrical rod is 628 cm^3. If its height is 20 cm, find the radius of its cross section. (Use $\pi = 3.14$)

Sol. Let radius of cross section of rod $= r$ cm

Height of cylindrical rod $= 20$ cm

Volume of cylindrical rod $= 628 \text{ cm}^3$

$\Rightarrow \quad \pi r^2 h = 628$

$\Rightarrow \quad 3.14 \times r^2 \times 20 = 628$

$\Rightarrow \quad r^2 = \frac{628 \times 100}{314 \times 20} = 10$

$\Rightarrow \quad r = \sqrt{10} \text{ cm} = 3.16 \text{ cm}$

$\therefore$ Radius of its cross section $= 3.16$ cm

Short Answer Type Questions II [3 Marks]

7. Water is flowing at the rate of 3 km/hour through a circular pipe of 20 cm internal diameter into a circular cistern of diameter 10 m and depth 2 m. In how much time will the cistern be filled?

Sol. Internal diameter of circular pipe $= 20$ cm

$\Rightarrow$ Internal radius $= r = \frac{20}{2} = 10 \text{ cm} = \frac{10}{100} = \frac{1}{10} \text{ m}$

Water is flowing at the rate of 3 km/hour $= 3000$ m/hour

Volume of water flowing in one hour $= \pi r^2 h = \pi\left(\frac{1}{10}\right)^2 \times 3000 \text{ m}^3$

Diameter of cistern $= 10$ m

Radius of cistern $= \frac{10}{2} = 5$ m

Depth of cistern $= 2$ m

Volume of water in cistern $= \pi r^2 h = \pi(5)^2 \times 2 \text{ m}^3$

Let the time taken to fill the cistern $= t$ hours

$\therefore \quad t = \frac{\text{Volume of water in cistern}}{\text{Volume of water flowing from pipe in 1 hour}}$

$= \frac{\pi(5)^2 \times 2}{\pi\left(\frac{1}{100}\right) \times 3000}$

$= 1\frac{2}{3}$ hours $= 1$ hour 40 minutes

8. The barrel of a fountain-pen cylindrical in shape, is 7 cm long and 5 mm in diameter. A full barrel of ink in the pen is used up on writing 330 words on an average. How many words would use up a bottle of ink containing one fifth of a litre ?

Sol. Length of cylindrical fountain pen = 7 cm = 70 mm

Diameter of cylindrical fountain pen = 5 mm

$\Rightarrow$ Radius of cylindrical fountain pen = $\frac{5}{2}$ mm

Volume of ink in cylindrical fountain pen = $\pi r^2 h = \pi\left(\frac{5}{2}\right)^2 \times 70$

$= \frac{22}{7} \times \frac{25}{4} \times 70 = 1375 \text{ mm}^3$

1375 mm^3 of ink is used to write 330 words

$\therefore$ $\frac{1375}{10^6}$ l of ink is used to write 330 words $(1l = 10^6 \text{ mm}^3)$

$\Rightarrow$ $\frac{1}{5}l$ of ink is used to write = $\frac{330 \times 1000000}{1375 \times 5}$ = 48000 words

9. Into a conical tent of radius 8.4 m and vertical height 3.5 m, how many full bags of wheat can be emptied, if space for the wheat in each bag is 1.96 m^3?

Sol. Radius of conical tent = 8.4 m

Height of conical tent = 3.5 m

$\therefore$ Capacity of the conical tent = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (8.4)^2 \times 3.5 = 258.72 \text{ m}^3$

Space of occupied by each bag of wheat = 1.96 m^3

$\therefore$ Number of bags = $\frac{\text{Capacity of the conical tent}}{\text{Space occupied by each bag of wheat}} = \frac{258.72}{1.96} = 132$

10. A conical tent is to accommodate 11 persons. Each person must have 4 square metres of space on the ground and 20 cubic metres of air to breathe. Find the height of the tent.

Sol. Space occupied by each person on the ground = 4 m^2

$\therefore$ Space occupied by 11 persons on the ground = 4 × 11 = 44 m^2

If r m be the radius of conical tent,

$\therefore$ Base area = πr^2

$\Rightarrow$ $\pi r^2 = 44$ ...(i)

Volume of air breathe by each person = 20 m^3

$\therefore$ Volume of air breathe by 11 persons = 11 × 20 = 220 m^3

If 'h' m be the height of conical tent,

$\therefore$ Volume of air in conical tent = $\frac{1}{3}\pi r^2 h$

$\Rightarrow$ $\frac{1}{3}\pi r^2 h = 220$ ...(ii)

Dividing (*ii*) by (*i*), we get

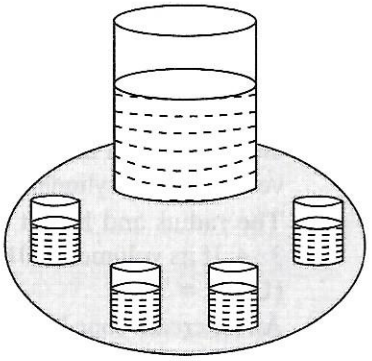
$$\frac{\frac{1}{3}\pi r^2 h}{\pi r^2} = \frac{220}{44}$$

$$\frac{h}{3} = 5 \Rightarrow h = 15 \text{ m}$$

$\therefore$ Height of the tent = 15 m

Long Answer Type Questions [4 Marks]

11. At a Ramzan Mela, a stall keeper in one of the food stalls has a large cylindrical vessel of base radius 15 cm filled up to a height of 32 cm with orange juice. The juice is filled in small cylindrical glasses (see figure) of radius 3 cm up to a height of 8 cm, and sold for Rs. 3 each. How much money does the stall keeper receive by selling the juice completely?



Sol. Radius of large cylindrical vessel = R = 15 cm

Height of large cylindrical vessel = H = 32 cm

$\therefore$ Volume of juice in large cylindrical vessel = $\pi R^2 H = \pi(15)^2 \times 32\ cm^3$

Radius of small cylindrical glass = r = 3 cm

Height of small cylindrical glass = h = 8 cm

$\therefore$ Volume of juice in each glass = $\pi r^2 h = \pi(3)^2 \times 8\ cm^3$

Number of glasses of juice that are sold = $\dfrac{\pi(15)^2 \times 32}{\pi(3)^2 \times 8} = 5 \times 5 \times 4 = 100$

Selling price of one glass = ₹ 3

$\therefore$ Total money received by selling 100 glasses = ₹ 3 × 100 = ₹ 300

12. A cylindrical road roller made of iron is 1 m long. Its inner diameter is 54 cm and the thickness of the iron sheet rolled into the road roller is 9 cm. Find the weight of the roller, if 1 cm^3 of iron weighs 8g. (Use $\pi = 3.14$)

Sol. Length of cylindrical road roller = h = 1 m = 100 cm

Inner diameter = 54 cm

$\Rightarrow$ Inner radius = $r = \dfrac{54}{2} = 27$ cm

Thickness of iron sheet = 9 cm

$\therefore$ Outer radius = $R = 27 + 9 = 36$ cm

$\therefore$ Volume of iron sheet = $\pi(R^2 - r^2) \times h = 3.14 \times [(36)^2 - (27)^2] \times 100$

$= 3.14 \times (1296 - 729) \times 100$

$= 3.14 \times 567 \times 100 = 178038\ cm^3$

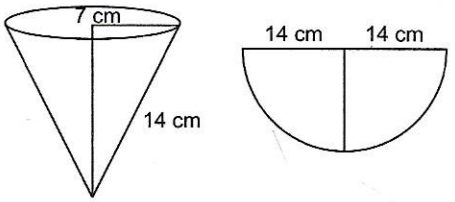
1 cm^3 of iron weighs = 8 g

$\therefore$ 178038 cm^3 of iron weighs = $8 \times 178038 = 1424304$ g = 1424.304 kg

$\therefore$ Weight of roller = 1424.304 kg

13. A semi-circular sheet of metal of diameter 28 cm is bent to form an open conical cup. Find the capacity of the cup.

Sol. According to question,



Circumference of conical cup = Circumference of semi-circle

$= \pi R = \pi \times 14$ cm

Let the radius of conical cup be r cm.

$\Rightarrow$ $2\pi r = 14\pi$

$\Rightarrow$ $r = \dfrac{14\pi}{2\pi} \Rightarrow r = 7$ cm

$\therefore$ Height of conical cup, $h = \sqrt{l^2 - r^2} = \sqrt{14^2 - 7^2}$

$= \sqrt{196 - 49} = \sqrt{147} = 12.12$ cm

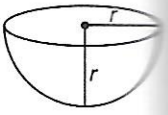
$\therefore$ Volume of the conical cup = $\dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3} \times \dfrac{22}{7} \times 7 \times 7 \times 12.12 = 622.16\ cm^3$

PRACTICE QUESTIONS BASED ON EXERCISES 13.6 AND 13.7

1. If the radius of a cylinder is doubled and height is halved, the volume will be doubled. Justify. [NCERT Exemplar]
2. A rectangular sheet of paper 44 cm × 18 cm is rolled along its length and a cylinder is formed. Find the volume of the cylinder. (Use $\pi = 22/7$)
3. The radius and height of a cone are in the ratio 3 : 4. If its volume is 301.44 cm^3, what is its radius? (Use $\pi = 3.14$)
4. An ice cream cone has the radius of base 2 cm. If its height is 10 cm, determine its volume.
5. A cylindrical pipe has inner diameter of 7 cm and water flows through it at 192.5 litres per minute. Find the rate of flow of water in km/hr.
6. If the radius of the base of a right circular cylinder is halved, keeping the height same, what is the ratio of the volume of the reduced cylinder to that of the original ?
7. A well with 10 m inside diameter is dug 14 m deep. Earth taken out of it and spread all around to a width of 5 m to form an embankment. Find the height of embankment.
8. What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?
9. Find the weight of a lead pipe 3.5 m long, if the external diameter of the pipe is 2.4 cm and the thickness of the lead is 2 mm and 1 cubic cm of lead weighs 11 g.
10. A sector of a circle of radius 12 cm has the angle 120°. It is rolled up so that two bounding radii are joined together to form a cone. Find the volume of the cone.
11. Water flows at the rate of 10 m/minute through a cylindrical pipe 5 mm in diameter. How long would it take to fill a conical vessel whose diameter of the base is 40 cm and depth 24 cm?
12. A lead pencil consists of a cylinder of wood with solid cylinder of graphite filled into it. The diameter of the pencil is 7 mm, the diameter of the graphite is 1 mm and the length of the pencil is 10 cm. Calculate the weight of the whole pencil, if the specific gravity of the wood is 0.7 g/cm^3 and that of the graphite is 2.1 g/cm^3. [HOTS]
13. A juice seller in a marriage party has a cylindrical vessel with $r = 25$ cm and $h = 40$ cm full of juice. He gives the same in small cylindrical glasses of radius 5 cm and height 10 cm. How many oranges are required for the bigger vessel to fill completely, if to fill one small glass two oranges are required?

Volume of a Hemisphere and a Sphere



- Volume of a Hemisphere $= \frac{2}{3}\pi r^3$, where r is the radius of the hemisphere.

- Volume of a Sphere $= \frac{4}{3}\pi r^3$, where r is the radius of the sphere.

SOLVED QUESTIONS BASED ON EXERCISE 13.8

Very Short Answer Type Questions [1 Mark]

1. The volume of two hemispheres are in the ratio 27 : 125. Find the ratio of their radii. [CBSE 2016]

Sol. Let r_1 and r_2 be the radii of two hemispheres.

$$\frac{\text{Volume of 1st sphere}}{\text{Volume of 2nd sphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$\frac{27}{125} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{5} \Rightarrow r_1 : r_2 = 3 : 5$$

2. If the radius of sphere is doubled. Find the ratio of volume of the new sphere to the original sphere.

Sol. Let radius of sphere be r

Volume of sphere $= \frac{4}{3}\pi r^3$

Radius of sphere is doubled

$\therefore$ Radius of new sphere $= 2r$

Volume of new sphere $= \frac{4}{3}\pi(2r)^3 = \frac{4}{3}\pi(8r^3)$

$$\text{Ratio of volumes} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(8r^3)} = \frac{1}{8}$$

3. The radius of sphere is 2r. Find its volume. **[NCERT Exemplar]**

Sol. Radius of sphere $= 2r$

Volume of sphere $= \frac{4}{3}\pi(2r)^3 = \frac{4}{3} \times 8\pi r^3 = \frac{32}{3}\pi r^3$

Short Answer Type Questions I [2 Marks]

4. The diameter of a sphere is decreased by 25%. Find its new volume.

Sol. Let radius of sphere be r

Diameter of sphere $= 2r$

Volume of sphere $= \frac{4}{3}\pi r^3$

Diameter of sphere decreased by 25%

New diameter $= (2r - 25\% \text{ of } 2r) = \left(2r - \frac{1}{4} \times 2r\right) = \frac{3}{4} \times 2r = \frac{3r}{2}$ $\left[25\% = \frac{25}{100} = \frac{1}{4}\right]$

New radius $= \frac{3r}{4}$

New volume of sphere $= \frac{4}{3}\pi\left(\frac{3r}{4}\right)^3 = \frac{4}{3} \times \frac{27}{64}\pi r^3 = \frac{9}{16}\pi r^3$

5. A solid sphere of radius 3 cm is melted and then recast into small spherical balls each of diameter 0.6 cm. Find the number of small balls thus obtained.

Sol. Radius of solid sphere $= 3$ cm

Volume of solid sphere $= \frac{4}{3}\pi(3)^3$

Diameter of small spherical ball $= 0.6$ cm

Radius of small spherical ball $= \frac{0.6}{2} = 0.3$ cm

Volume of small spherical ball $= \frac{4}{3}\pi(0.3)^3$ cm^3

$$\text{Number of small spherical balls} = \frac{\frac{4}{3}\pi(3)^3}{\frac{4}{3}\pi(0.3)} = \frac{3 \times 3 \times 3}{0.3 \times 0.3 \times 0.3} = 1000$$

6. The diameter of a sphere is 42 cm. It is melted and drawn into a cylindrical wire of 28 cm diameter. Find the length of the wire.

Sol. Diameter of sphere = 42 cm

Radius of sphere = $\frac{42}{2}$ = 21 cm

Diameter of cylindrical wire = 28 cm

Radius of cylindrical wire = $\frac{28}{2}$ = 14 cm

Let h be the length of cylindrical wire

Now, volume of sphere = volume of cylindrical wire

$$\frac{4}{3}\pi(21)^3 = \pi(14)^2 \times h$$

$$\Rightarrow \quad h = \frac{4}{3} \times \frac{21 \times 21 \times 21}{14 \times 14} = 63 \text{ cm}$$

7. A hemispherical bowl of internal diameter 36 cm contains a liquid. This liquid is to be filled in cylindrical bottles of radius 3 cm and height 6 cm. How many bottles are required to empty the bowl ?

Sol. Internal radius of cylindrical bowl = $\frac{36}{2}$ = 18 cm

Radius of cylindrical bottle = 3 cm

Height of cylindrical bottle = 6 cm

Let the number of required bottles be n

Volume of hemispherical bowl = n(volume of cylindrical bottles)

$$\frac{2}{3}\pi(18)^3 = n[\pi(3)^2 \times 6]$$

$$n = \frac{2 \times 18 \times 18 \times 18}{3 \times 9 \times 6} = 72$$

Short Answer Type Questions II [3 Marks]

8. Solid spheres of diameter 4 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 12 cm and the water rises by 24 cm, find the number of solid spheres dropped in the water.

Sol. Diameter of solid sphere = 4 cm

Radius of solid spheres = $\frac{4}{2}$ = 2 cm

Let number of spheres dropped in water be n

Volume of n spheres = $n\left[\frac{4}{3}\pi(2)^3\right]$

Diameter of cylindrical beaker = 12 cm

Radius of cylindrical beaker = $\frac{12}{2}$ = 6 cm

Height upto which the water level rises = 24 cm

Volume of water rises = $\pi(6)^2 \times 24$ cm^3

As per question,

$$n\left[\frac{4}{3}\pi(2)^3\right] = \pi \times 36 \times 24$$

$$n = \frac{36 \times 24 \times 3}{4 \times 8} = 81$$

9. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 3.5 cm and height 3 cm. Find the number of cones so formed.

Sol. Diameter of metallic sphere = 21 cm

Radius of metallic sphere = $\frac{21}{2}$ cm

Volume of metallic sphere = $\frac{4}{3}\pi\left(\frac{21}{2}\right)^3$ cm^3

Diameter of cone = 3.5 cm

Radius of cone = $\frac{3.5}{2}$ cm

Height of cone = 3 cm

Volume of cone = $\frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3$ cm^3

Let number of cones be n

$$\text{Volume of } n \text{ cones} = n\left[\frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3\right] \text{ cm}^3$$

As per question, volume of sphere = volume of n cones

$$\frac{4}{3}\pi\left(\frac{21}{2}\right)^3 = \frac{n}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3$$

$$\Rightarrow \quad n = \frac{4 \times 21 \times 21 \times 21 \times 3 \times 2 \times 2}{3 \times 2 \times 2 \times 2 \times 3 \times 3.5 \times 3.5} = 504$$

10. A hemispherical bowl of internal diameter 30 cm contains some liquid. This liquid is to be filled into cylindrical shaped bottles each of diameter 5 cm and height 6 cm. Find the number of bottles necessary to empty the bowl.

Sol. Internal diameter of hemispherical bowl = 30 cm

Internal radius of hemispherical bowl = R = $\frac{30}{2}$ = 15 cm

$\therefore$ Volume of liquid in hemispherical bowl = $\frac{2}{3}\pi R^3 = \frac{2}{3}\pi \times (15)^3$ cm^3

Diameter of cylindrical bottle = 5 cm

Radius of cylindrical bottle = $r = \frac{5}{2}$ cm

Height of cylindrical bottle = h = 6 cm

Let number of cylindrical shaped bottles be n.

Volume of n cylindrical shaped bottles = $n[\pi r^2 h] = n\left[\pi\left(\frac{5}{2}\right)^2 \times 6\right]$ cm^3

As per question, volume of liquid in hemispherical bowl and n cylindrical bottle remains same.

$\therefore$ $$n\left[\pi \times \frac{25}{4} \times 6\right] = \frac{2}{3}\pi \times (15)^3$$

$$n = \frac{2 \times 15 \times 15 \times 15 \times 4}{3 \times 25 \times 6} = 60$$

$$n = 60$$

$\therefore$ Required number of bottles to empty the bowl = 60

Long Answer Type Questions [4 Marks]

11. The volumes of two spheres are in the ratio 64 : 27. Find their radii, if the sum of their radii is 21 cm.

Sol. Let r_1 cm be the radius of 1st sphere and r_2 cm be the radius of 2nd sphere.

Also, $r_1 + r_2 = 21$ cm

$$\frac{\text{Volume of 1st sphere}}{\text{Volume of 2nd sphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3}$$

$$\Rightarrow \quad \frac{64}{27} = \frac{r_1^3}{r_2^3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

$$\Rightarrow \quad r_1 = \frac{4}{3}r_2$$

Putting $r_1 = \frac{4}{3}r_2$ in (i), we get

$$\frac{4}{3}r_2 + r_2 = 21$$

$$\Rightarrow \quad \frac{7r_2}{3} = 21$$

$$\Rightarrow \quad r_2 = \frac{21 \times 3}{7} = 9 \text{ cm}$$

$$\Rightarrow \quad r_1 = 21 - 9 = 12 \text{ cm}$$

$\therefore$ Radius of 1st sphere = 12 cm

Radius of 2nd sphere = 9 cm

12. The largest sphere is carved out of a cube of side 7 cm. Find the volume of the sphere. [HOTS]

Sol. Side of cube = 7 cm

Diameter of sphere = Side of cube = 7 cm

Radius of sphere = $r = \frac{7}{2}$ cm

$\therefore$ Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{7}{2}\right)^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7 \times 7 \times 7}{2 \times 2 \times 2} = 179.67 \text{ cm}^3$$

13. The total cost of making a spherical ball is ₹ 33,957 at the rate of ₹ 7 per cubic metre. What will be the radius of this ball?

Sol. Let radius of the ball be r m.

$\therefore$ Volume of spherical ball $= \frac{4}{3}\pi r^3$

Total cost of making a spherical ball at the rate of ₹ 7 per m^3 = ₹ 33957

$\therefore$ Volume of spherical ball $= \frac{33957}{7} = 4851 \text{ m}^3$

Now, $\frac{4}{3}\pi r^3 = 4851$

$$\Rightarrow \quad r^3 = \frac{4851 \times 7 \times 3}{4 \times 22} = \frac{441 \times 21}{8} = \left(\frac{21}{2}\right)^3$$

$$\Rightarrow \quad r^3 = \left(\frac{21}{2}\right)^3 \Rightarrow r = \frac{21}{2} = 10.5 \text{ m}$$

$\therefore$ Radius of spherical ball = 10.5 m

PRACTICE QUESTIONS BASED ON EXERCISE 13.8

1. Find the volume of metal used to construct a hollow metallic sphere of internal and external diameters as 10 cm and 13 cm respectively (Use $\pi = 3.14$)
[CBSE 2016]
2. A spherical metallic shell with 10 cm external diameter weighs $1789\frac{1}{3}$ g. Find the thickness of the shell if the density of metal is 7 g/cm^3.
[HOTS; CBSE 2016]
3. A metallic sphere of diameter 12.6 cm is melted to make a right circular cone of height 25.2 cm. Calculate the radius of cone.
4. Find the volume of a sphere whose diameter is 7 cm. (Use $\pi = \frac{22}{7}$) **[CBSE 2010]**
5. A spherical cannon ball, 28 cm in diameter is melted and cast into a right circular conical mould the base of which is 35 cm in diameter. Find the height of the cone, correct to two places of decimal.
6. A hemisphere lead of radius 8 cm is cast into a right circular cone of base radius 6 cm. Determine the height of the cone, correct to two places of decimal.
7. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of small cones, each of diameter 7 cm and height 3 cm. Find number of cones so formed.
8. A vessel in the form of a hemispherical bowl is full of water. Its contents are emptied in a right circular cylinder. The internal radii of the bowl and the cylinder are 3.5 cm and 7 cm respectively. Find the height to which the water will rise in the cylinder.
9. A cylindrical tub of radius 16 cm contains water to a depth of 30 cm. A spherical iron ball is dropped into the tub and thus level of water is raised by 9 cm. What is the radius of the ball ?
10. A measuring jar (cylindrical shape) of internal diameter 10 cm is partially filled with water. Four equal spherical balls of diameter 2 cm each are dropped in it and they sink down in water completely. What will be the change in the level of water in the jar ?
11. Lead spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and water rises by 40 cm, find the number of lead spheres dropped in the water.

Value Based Questions

1. A cancer detective centre is going to develop in our city of cubicel shape having length 600 m, breadth 500 m and height 400 m.
   (*i*) Calculate its total area.
   (*ii*) What concept is derived from this activity?
2. An old age centre of conical structure having radius 21 m and lateral height 147 m is being set up to provide basic needs to our senior citizens.
   (*i*) Calculate its lateral surface area.
   (*ii*) What ideas are being promoted here?
3. An aquarium in a DDA park is of hemi-spherical structure, and the radius is 2.1 km.
   (*i*) Calculate the total surface area.
   (*ii*) What ideas are being promoted here?

INTEGRATED EXERCISE

Very Short Answer Type Questions [1 Mark]

1. A gift kept in cubical box with edge 10 cm is to be packed with a hand made paper. Find the amount of paper used to pack the gift. **[CBSE 2016]**
2. The volume of a cylinder is 448π cm^3 and height 7 cm. Find its total surface area.
3. A granary is in the shape of a cuboid of size 8 m × 6 m × 3 m. A bag of grain occupies a space of 0.65 m^3. How many bags can be stored in the granary?
4. The diagonal of a cube is $\sqrt{12}$ cm. Find its volume. **[CBSE 2014]**
5. The perimeter of one face of cube is 20 cm. Find its volume. **[CBSE 2014]**

Short Answer Type Questions I [2 Marks]

6. If the length of the diagonal of a cube is $6\sqrt{3}$ cm, then the length of the edge of the cube is 3 cm. Justify. **[NCERT Exemplar]**
7. The length of a hall is 20 m and width 16 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height and the volume of the hall.
8. Navinchal received a white card board cube of side 0.5 m. He did not like the raw cube and decided to put the blue stars on the surfaces, without folding. Area of each star is 40 cm^2. Calculate the area of the remaining if he put 372 blue stars in total on the surface.

9. A cone and a hemisphere have equal bases and equal volumes. Find the ratio of their heights.
10. Volume of a cube is 64,000 m^3. Find the cost of painting its total surface at the rate of ₹ 5 per m^2. **[CBSE 2010]**

Short Answer Type Questions II [3 Marks]

11. A gift pack is in the shape of cuboid of dimensions 1.5 m, 0.75 m, 0.5 m. Sanya did not like the paper wrapped on the gift pack, so she removed it and wrapped the gift pack with a paper of her choice but left the base 7.5 m × 0.75 m, which she said to fill with her name. Calculate the difference in areas of wrapping papers used. **[HOTS]**
12. The surface areas of a sphere and a cube are equal. Prove that their volumes are in the ratio $1 : \sqrt{\pi/6}$.
13. The length of a cold storage is double its breadth. Its height is 3 metres. The area of four walls excluding a door of dimension 4 m × 2 m is 100 m^2. Find its volume.
14. If V is the volume of cuboid of dimensions a, b, c and S is the surface area, the prove that $\frac{1}{V} = \frac{2}{S}\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$.
15. A rectangular water tank of base 11 m × 6 m contains water upto a height of 5 m. If the water in the tank is transferred to a cylindrical tank of radius 3.5 m, find how high will the water level be in this tank. (Use $\pi = 22/7$)
16. The length of a cuboid is 16 m and its breadth and height are in the ratio 3 : 2. If the volume of the cuboid is 1536 cu.m, find the breadth and height of the cuboid and the total surface area of the cuboid. **[CBSE 2015]**
17. The length, breadth and height of the room are 5 m, 4 m and 3 m respectively. Find the total cost of white washing the ceiling at the rate of ₹ 12 per m^2 and four walls at the rate of ₹ 10 per m^2. **[CBSE 2016]**
18. 50 circular plates, each of radius 7 cm and thickness $\frac{1}{2}$ cm, are placed one above another to form a solid right circular cylinder. Find the total surface area and the volume of the cylinder so formed.
19. The cost of painting to the total outside surface of a closed cylindrical oil tank at 60 paise per square dm is ₹ 237.60. The height of the tank is 6 times the radius of the base of the tank. Find its volume.
20. Find the cost of sinking a tubewell 280 m deep having diameter 3 m at the rate of ₹ 3.60 per m^3. Find the cost of cementing its inner curved surface at ₹ 2.50 per m^2.
21. The volume of a cylinder is 5544 cm^3 and its height is 16 cm. Find its radius and hence its curved surface area.
22. How many cubic metres of earth must be dug out to sink a well 24 m deep and of diameter 7 m? Also, find the cost of plastering the inner curved surface at ₹ 3 per square metre.
23. A solid iron rectangular block of dimensions 4.4 m, 2.6 m and 1 m is cast into a hollow cylinder pipe of internal radius 30 cm and thickness 5 cm. Find the length of the pipe.
24. A heap of wheat is in the form of a cone of diameter 10.5 m and height 3 m. Find its volume. How much canvas cloth is required to just cover the heap?
25. A cylinder and a cone have equal radii of their bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the radius of each is to the height of each as 3 : 4.
26. A classroom is 10 m long, 6.4 m wide and 5 m high. If each student be given 1.6 m^2 of the floor area, how many students can be accommodated in the room? How many cubic metres of air would each student get? **[CBSE 2010; HOTS]**
27. A metallic sheet is of the rectangular shape with dimensions 42 cm and 36 cm from each one of its corner, a square of 6 cm is cut off. An open box is made of the remaining sheet. Find the volume of the box.
28. Find the volume of the largest right circular cone that can be cut off from a cube whose edge is 9 cm. (Use $\pi = 22/7$)

Long Answer Type Questions [4 Marks]

29. A cone of height 24 cm has a curved surface area of 550 cm^2. Find its volume.
30. A rectangular sheet of paper 30 cm × 18 cm can be transformed into the curved surface of a right circular cylinder in two ways either by rolling the paper along its length or by rolling it along its breadth. Find the ratio of the volumes of the two cylinders thus formed. **[HOTS]**
31. Height of a solid cylinder is 10 cm and diameter is cm. Two identical conical holes have been made

from its both ends. The diameter of the holes is 6 cm and height 4 cm. Find (*i*) the volume of the cylinder, (*ii*) volume of one conical hole (*iii*) volume of remaining solid.

32. The height of a cone is 24 cm and the diameter of its base is 14 cm. Find the slant height, volume, curved surface area and the total surface area of the cone.

33. The internal and external diameters of a hollow hemispherical vessel are 24 cm and 25 cm respectively. The cost to paint 1 cm^2 of the surface is ₹ 0.05. Find the total cost to paint the vessel all over.

(Use $\pi = \frac{22}{7}$)

34. Water in a canal 30 dm wide and 12 dm deep is flowing with a velocity of 20 km/hr. How much area will it irrigate in 30 minutes, if 9 cm of standing water is desired?

35. A rectangular water reservoir is 10.8 m by 3.75 m at the base. Water flows into it at the rate of 18 m/s through a pipe having the cross section 7.5 cm × 4.5 cm. Find the height to which the water will rise in the reservoir in 30 minutes.

36. To construct a wall 24 m long, 0.4 m thick and 6 m high, bricks of dimensions 25 cm × 16 cm × 10 cm each are used. If the mortar occupies one-tenth of the volume of the wall, find the number of bricks used.

37. A storage tank is in the form of a cube. When it is full of water, the volume of water is 15.625 m^3. If the present depth of water is 1.3 m, find the volume of water already used from the tank. **[NCERT Exemplar]**

38. A solid cylinder has a total surface area 462 sq. cm. Its curved surface area is one-third of the total surface area. Find the volume of the cylinder.

39. Water is flowing at the rate of 7 m/s through a circular pipe whose internal diameter is 2 cm into a cylindrical tank the radius of whose base is 40 cm. Determine the increase in the water level in 1/2 hr.

40. The pillars of a temple are cylindrical shaped. If each pillar has a circular base of radius 25 cm and height 10.5 m, then find the quantity of concrete mixture used to build 30 such pillars. Also find the cost of concrete mixture at the rate of Rs. 250 per $metre^3$. (Take $\pi = 22/7$)

41. How many spherical lead shots each 4.2 cm in diameter can be obtained from a rectangular solid of lead with dimensions 66 cm, 42 cm and 21 cm? (Use $\pi = 22/7$)

42. A solid metallic sphere of diameter 21 cm is melted and recasted into a number of smaller cones, each of diameter 3.5 cm and height 3 cm. Find the number of cones so formed.

43. The sum of the radius of the base and height of a cylinder is 37 m. If the total surface area of the solid cylinder is 1628 m^2, find the volume of cylinder.

44. A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, find the uniform thickness of the cylinder.

ASSESS YOURSELF

1. Find the length of the longest rod that can be placed in a room of dimensions 30 m × 24 m × 18 m.

2. Four cubes each of side 5 cm are joined end to end. Find the surface area of the resulting cuboid.

3. Find the volume and total surface area of a cube of edge 15 cm.

4. Two cubes have volumes in the ratio 27 : 64. Find the ratio of their surface areas. **[HOTS]**

5. If each edge of a cube is increased by 25%, then find the percentage increase in its surface area. **[HOTS]**

6. A cylinder is 3 m high and the circumference of its base is 22 m. Find its curved surface area.

7. A hollow cylinder pipe has inner circumference 44 dm and outer circumference 45 dm. Find the cost of painting it from both sides at ₹ 2 per m^2, if its length is 3.5 m.

8. A roller 150 cm long has a diameter 70 cm. To level a playground, it takes 750 complete revolutions. Determine the cost of levelling the playground at the rate of 75 paise per square metre.

9. A building has 8 pillars each having diameter 50 cm and height 3.5 m. Find the cost of painting their curved surfaces at the rate of ₹ 12.50 per square metre.

10. In a cylinder, if the radius is doubled and height is halved, then find its new curved surface area. **[NCERT Exemplar]**

11. A cylindrical roller 2.5 m in length, 1.75 m in radius when rolled on a road was found to cover the area of 5500 m^2. How many revolutions did it make? **[NCERT Exemplar]**

12. Circumference of the base of a right circular cone is 88 cm. If height of the cone is 10 cm then, find its volume. **[CBSE 2010]**

13. The radius of a cone is 5 cm and height is 12 cm. Find curved surface area and volume of the cone. (Use $\pi = 3.14$)

14. Find the total surface area of cube. If its volume is 3375 cm^3.

15. Find the volume and surface area of a sphere of radius 4.3 cm. (Use $\pi = 3.14$)

16. The internal and external radius of a hollow hemispherical vessel are 12 cm and $\frac{25}{2}$ cm respectively.
The cost of painting 1 sq. cm of surface is ₹ 0.07. Find the total cost to paint the vessel.

17. From the given cuboid of dimensions, $l = 5$ cm, $b = 4$ cm and $h = 2$ cm, how many cubes of edge 2 cm can be cut out of it?

18. A solid cube is cut into two cuboids of equal volumes. Find the ratio of the total surface area of the given cube and that of one of the cuboids.

19. A cuboidal tank is 6 m long, 5 m wide and 4.5 m deep. How many litres of water it can hold?

20. Volume of a cube is 5832 m^3. Find the cost of painting its total surface area at the rate of ₹ 3.50 per m^2. **[CBSE 2010]**

21. A storage tank is in the form of a cube. When it is full of water, the volume of water is 42.875 m^3. If the present depth of water is 2.3 m, find the volume of water already used from the tank.

22. The size of the base of a cane full of kerosene is 20 cm × 20 cm and its height is 45 cm. The kerosene of this cane is poured into another cane having base of size 25 cm × 15 cm. Determine the height of the kerosene in the second cane.

23. Volume of a right circular cone is $\frac{2200}{7}$ cubic cm and its diameter is 10 cm. Find slant height and also the curved surface area of cone in terms of π.

24. The volume of the space inside a right circular conical tent is $138\frac{2}{7}$ m^2 and its vertical height is 4 m. Find the canvas required to make the tent and also, find the cost of the canvas at the rate of ₹ 120 per m^2 (Take $\sqrt{33} = 5.74$).

25. If the heights of two cones are in the ratio 1 : 3 and their diameters are in the ratio 3 : 5, find the ratio of their volumes.

26. How many spherical bullets can be made out of a solid cube of lead whose edge measures 44 cm, each bullet being 4 cm in diameter? (Use $\pi = \frac{22}{7}$)

27. Solid spheres of diameter 6 cm are dropped into a cylindrical beaker containing some water and are fully submerged. If the diameter of the beaker is 18 cm and the water rises by 40 cm, find the number of solid spheres dropped in the water.

28. Find the volume of a sphere whose surface area is 154 cm^2.